Application of the Spectral Matrix in Wave Vector Analysis

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Introduction

The Spectral Matrix Method (SMM) is a class of methods used to determine the direction of travel of waves propagating through a magnetic field. These directions are called wave vectors and they can be calculated using tri-axial magnetometer data. This project aims to replicate the results of another researcher, Ulrich Taubenschuss, who quantified SMM performance using simulated data.

Method

Tri-axial magnetometers measure magnetic field strength in three orthogonal axes as a function time. Waves that propagate through the field can be locally approximated as plane waves and an idealized, noise-less signal might look like the plot below with each color representing an axis.

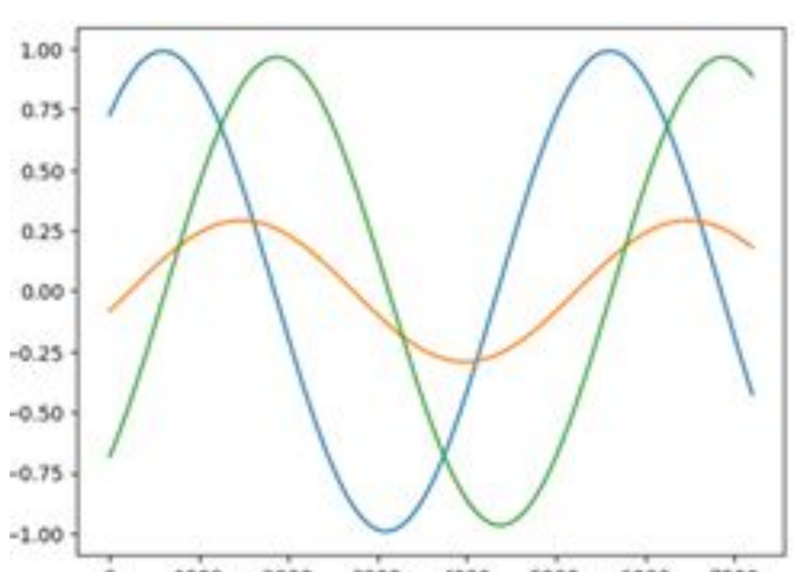

Figure 1: Idealized Plane Wave

In reality, noise pervades all measurements and if the signal to noise ratio is really low, the data might look like the simulated time series below.

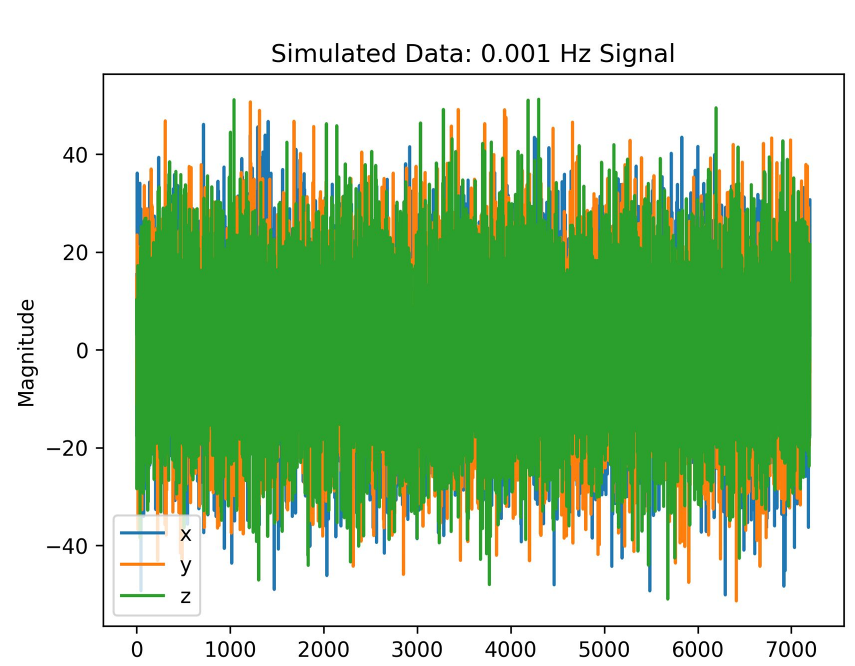

Figure 2: Plane Wave with White Noise

SMM is used to find the wave vectors of plane waves buried in the noise. The Spectral Matrix is constructed as covariance a matrix containing correlation terms for the wavelet transforms of each magnetometer axis. Mathematical notation is provided below where J is the Spectral Matrix.

$$H_x = \mathrm{CWT}[B_x(\mathrm{t})]$$
$$H_y = \mathrm{CWT}[B_y(\mathrm{t})]$$
$$H_z = \mathrm{CWT}[B_z(\mathrm{t})]$$

$$J = \begin{pmatrix} H_x H_x^* & H_x H_y^* & H_x H_z^* \\ H_y H_x^* & H_y H_y^* & H_y H_z^* \\ H_z H_x^* & H_z H_y^* & H_z H_z^* \end{pmatrix}$$

The components of the spectral matrix are visualized in the sample plots below.

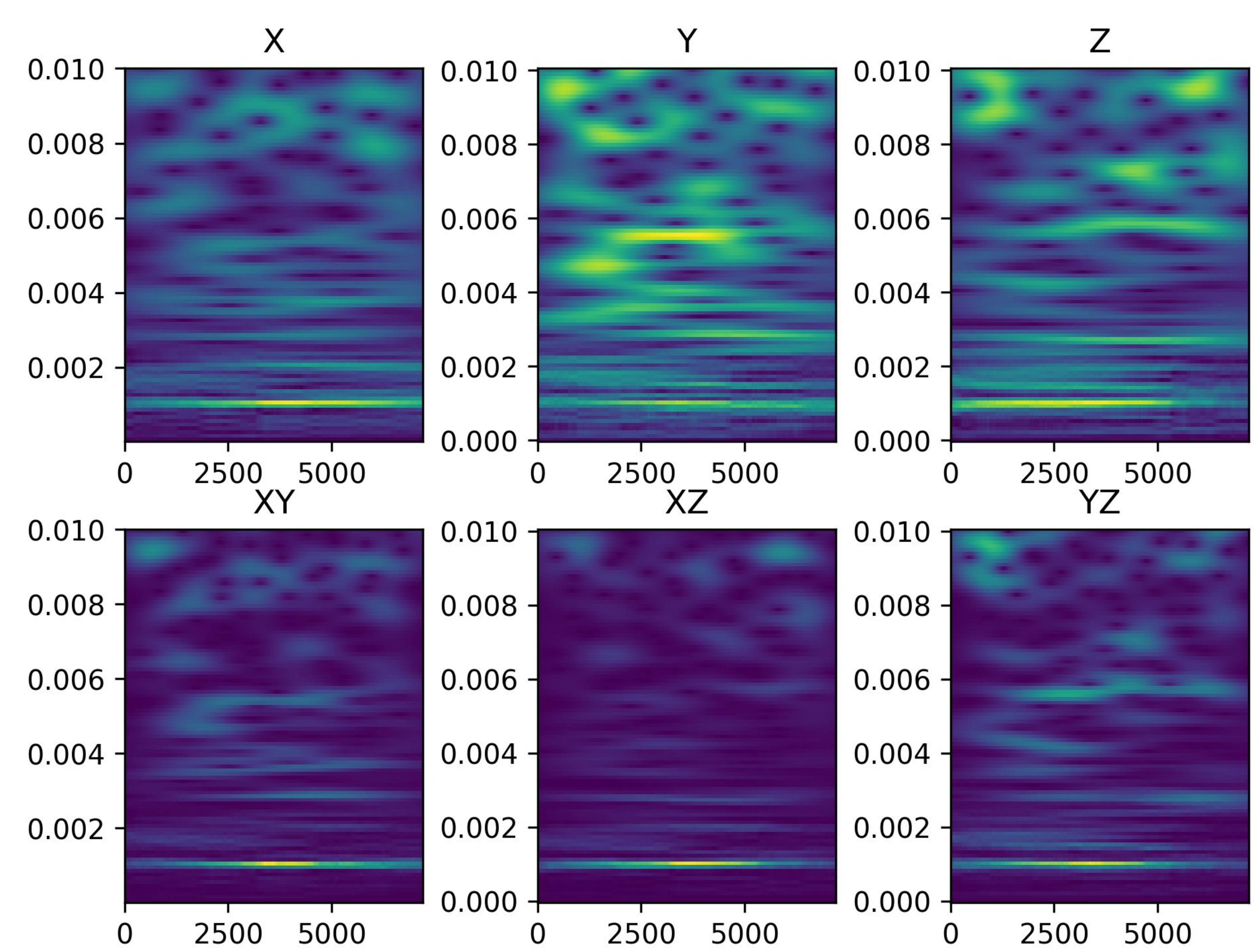

Figure 3: Frequency-Time Plots

These matrices are processed using Singular Value Decomposition, which produces the unit wave vector of a signal within the searched frequency band.

The above process was implemented in Python and put through a series of performance tests to see how reliably wave vectors can be found given different noise levels. The results are seen in figures 4 and 5. Tests were performed using simulated data.

Results

A plane wave was generated at an angle of 20 degrees to the x-axis and over multiple trials, various levels of white noise were added. SMM was used to compute the angle at each noise level. As the SNR increases, the angle estimation gets better and produces the curve provided by Taubenschuss below.

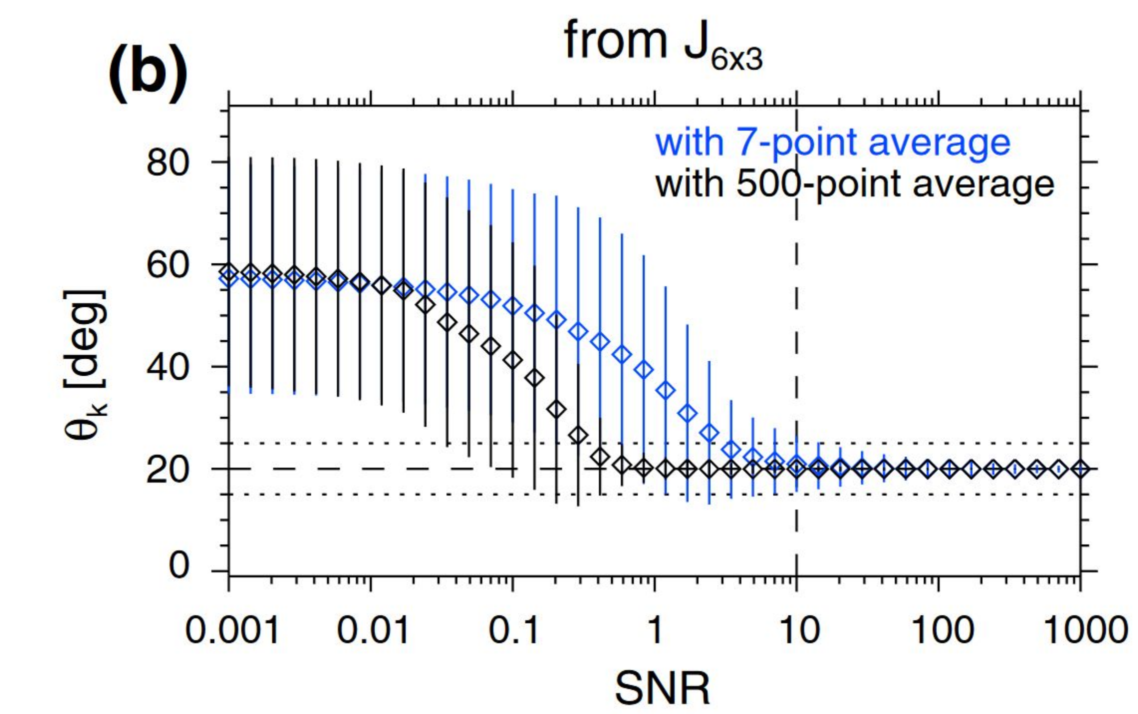

Figure 4: Taubenschuss Theta Plot [2]

When performing the same test, we got the plot below. Both the curve and error bars are in good agreement with those above. This confirms that SMM performs as well as claimed.

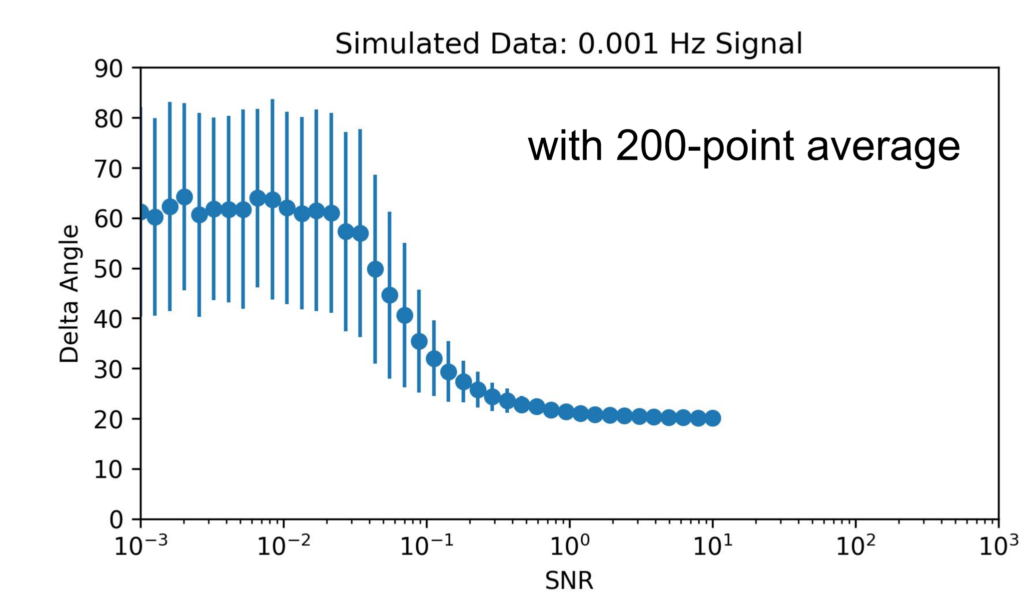

Figure 5: Our Theta Plot

Future work will involve calculating the magnitude of wave vectors, as SMM only gets the direction.

References

[1] Santolík, O., et al. "Singular Value Decomposition Methods for Wave Propagation Analysis." Radio Science, vol. 38, no. 1, 2003, https://doi.org/10.1029/2000rs002523.

[2] Taubenschuss, Ulrich, and Ondřej Santolík. "Wave Polarization Analyzed by Singular Value Decomposition of the Spectral Matrix in the Presence of Noise." Surveys in Geophysics, vol. 40, no. 1, 2018, pp. 39–69., https://doi.org/10.1007/s10712-018-9496-9.